STAT 2593

Lecture 015 - The Poisson Probability Distribution

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The Poisson Probability Distribution

Learning Objectives

1. Understand the poisson distribution, its use cases, and its properties.

2. Understand the use of a poisson process.

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  - The values of x can be any non-negative integer, $0, 1, 2, \ldots$.
  - We have that $E[X] = \mu$ and that $\text{var}(X) = \mu$.

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  - This is well described by a Poi(αt) distribution.
- Note, we assume that there is concordance with the rate α and the time interval t.

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- Specifically, if we take $n \to \infty$ and $p \to 0$ such that $np = \mu$, then this is well approximated by a poisson distribution.
  - You will likely see the rule-of-thumb of $n > 50$ and $np < 5$, but this is mostly just guidance!

Summary

- The poisson distribution counts the occurrences of events over an interval of time.
- The poisson distribution has a well-defined pmf, expectation, and variance.
- The poisson process directly captures the arrivals of events in sequence, under set assumptions.
- The poisson distribution can be viewed as the limit to a binomial distribution with more and more trials.